Dynamical generation of gauge groups in the massive Yang-Mills-Chern-Simons matrix model

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It has been known that the dynamics of k coincident D-branes in string theory is described effectively by U(k) Yang-Mills theory at low energy. While these configurations appear as classical solutions in matrix models, it was not clear whether it is possible to realize the $k \neq 1$ case as the true vacuum. The massive Yang-Mills-Chern-Simons matrix model has classical solutions corresponding to all the representations of the SU(2) algebra, and provides an opportunity to address the above issue on a firm ground. We investigate the phase structure of the model, and find in particular that there exists a parameter region where O(N) copies of the spin-1/2 representation appear as the true vacuum, thus realizing a nontrivial gauge group dynamically. Such configurations are analogous to the ones that are interpreted in the BMN matrix model as coinciding transverse 5-branes in M-theory.

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Introduction.— The discovery of D-branes as classical solutions in string theory suggested an interesting scenario that we are actually living on a kind of D-brane embedded in a higher-dimensional space-time ("the brane-world scenario"). The low energy effective theory of a D-brane is given by a U(1) gauge theory, and in the case of k coincident D-branes, the gauge group enhances to U(k). Along this line, there are a lot of activities in the search of a *perturbatively* stable brane configuration which realizes the Standard Model at low energy.

On the other hand, it is also possible that our world is realized as the true *nonperturbative* vacuum of superstring/M theories. Such an issue may be addressed by studying the matrix models [1,2], which are proposed as nonperturbative formulations of superstring/M theories. In particular the IIB matrix model [2] can be obtained by dimensionally reducing the 10-dimensional $\mathcal{N} = 1$ super Yang-Mills theory to a point. The space-time should come out dynamically as the four extended directions in the eigenvalue distribution of the 10 bosonic matrices [3]. The mechanism that realizes the brane world in this way has been studied in Refs. [4,5], and by now there are certain evidences that indeed *four*-dimensional space-time is generated dynamically [6–9].

The possibility of obtaining the U(k) gauge group dynamically has been discussed in Ref. [10]. There it was claimed that if the eigenvalues of the ten bosonic matrices form clusters of size k, the low energy effective theory should have the U(k) gauge symmetry. However, whether such clustering really occurs as a dynamical property of the IIB matrix model remains unclear.

In the series of papers [11–13], we discussed the same issue in matrix models with a cubic term. In these models the fuzzy spheres [14] appear as classical solutions [15] and play the role of D-branes [16]. One of the advantages of studying fuzzy spheres in the present context is that they are solutions even for finite matrices, and therefore one can study their dynamical properties by well-defined perturbative calculations. (See Refs. [17] for other motivations for fuzzy spheres.) The expansion around the k coincident fuzzy spheres yields a U(k) gauge theory on a noncommutative geometry [15]. Therefore, by comparing the free energy for various fuzzy sphere configurations, we may discuss the dynamical generation of gauge groups. However, in the simplest Yang-Mills-Chern-Simons matrix model [11,12], as well as in its higher-dimensional extensions [13], which accommodate four-dimensional fuzzy manifolds, the gauge group generated dynamically turned out to be U(1).

In this letter we show that in fact one can obtain gauge groups of higher rank dynamically by adding a "mass term" [18] to the matrix models. A similar model is known to appear in M-theory on a plane wave background [19]. Although we consider that such a phenomenon occurs in more general models, here we discuss the "massive" Yang-Mills-Chern-Simons matrix model for simplicity.

The model and its classical solutions.— The model we consider in this letter is defined by the action

$$S = N\,\alpha^4 \mathrm{tr}\left\{ -\frac{1}{4}\,[A_\mu, A_\nu]^2 + \frac{2}{3}\,i\,\epsilon_{\mu\nu\lambda} A_\mu A_\nu A_\lambda + \frac{1}{2}\,\rho^2\,(A_\mu)^2 \right\}\,, \qquad (1)$$

where A_μ ($\mu = 1, 2, 3$) are $N \times N$ Hermitian matrices. (Rewriting the action in terms of $\mathcal{A}_\mu = \alpha\,A_\mu$, one obtains the model of Ref. [11] plus a mass term $\frac{1}{2}\,M^2\,(\mathcal{A}_\mu)^2$ with $M = \alpha\rho$.) The classical equation of motion

$$-[A_\nu,[A_\mu,A_\nu]] + i\,\epsilon_{\mu\nu\lambda}\,[A_\nu,A_\lambda] + \rho^2 A_\mu = 0 \qquad (2)$$

has a class of perturbatively stable solutions given by $A_\mu = X_\mu \equiv \chi\, L_\mu$ for $0 < \rho < \frac{1}{\sqrt{2}}$, where L_μ ($\mu = 1,2,3$) is an arbitrary N-dimensional representation of the SU(2) algebra and $\chi \equiv \frac{1}{2}\left(1 + \sqrt{1 - 2\rho^2}\right)$. Using the SU($N$) symmetry, the representation matrix L_μ can be brought into the block-diagonal form with k_i blocks of the n_i-dimensional irreducible representation $L_\mu^{(n_i)}$ satisfying $n_1 < n_2 < \cdots < n_s$ and $\sum_{i=1}^{s} n_i\, k_i = N$. (The trivial solution $A_\mu = 0$ may be regarded as a particular case in which one chooses N copies of the representation $L_\mu^{(1)} = 0$.) Since the Casimir operator for each irreducible block is given by $\left(L_\mu^{(n)}\right)^2 = \frac{1}{4}(n^2-1)\,\mathbf{1}_n$, the configuration X_μ may be viewed as a collection of k_i coincident fuzzy spheres with the radius $\frac{1}{2}\chi\sqrt{(n_i)^2 - 1}$, and the classical action can be evaluated as

$$S_{\rm cl} = \frac{1}{4}\,N\,\alpha^4\, f(\chi)\sum_{i=1}^{s} k_i\, n_i \left\{(n_i)^2 - 1\right\} , \qquad (3)$$

where $f(\chi) = \frac{1}{2}\chi^4 - \frac{2}{3}\chi^3 + \frac{1}{2}\rho^2\chi^2$.

Since $f(\chi)$ is positive (negative) for $\rho > \frac{2}{3}$ ($\rho < \frac{2}{3}$), the minimum of $S_{\rm cl}$ is given by the single fuzzy sphere $A_\mu = \chi\, L_\mu^{(N)}$ for $\rho < \frac{2}{3}$, and by the trivial solution $A_\mu = 0$ for $\rho > \frac{2}{3}$. These configurations describe the true vacuum in the large-α limit where quantum fluctuations are suppressed. As we decrease α, however, the one-loop effects become non-negligible, and it is possible that some other solution describes the true vacuum.

One-loop calculation.— Let us evaluate the partition function $Z = \int dA\, e^{-S}$ around a general solution $A_\mu = X_\mu$ at the one-loop level. The measure for the path integral is defined by $dA = \prod_{\mu=1}^{3}\prod_{a=1}^{N^2} dA_\mu^a$, where $A_\mu = \sum_{a=1}^{N^2} A_\mu^a\, t^a$ with t^a being the generators of U(N) normalized by $\mathrm{tr}\,(t^a\, t^b) = \delta_{ab}$.

For solutions other than $A_\mu = 0$, we need to fix the gauge since there are flat directions corresponding to the transformation $A_\mu \mapsto A_\mu^g \equiv g\, A_\mu\, g^\dagger$, where g is an element of the coset space $H \equiv \mathrm{U}(N)/\prod_{i=1}^{s}[\mathrm{U}(k_i)]$. We take the gauge fixing condition

$$i\,[X_\mu, A_\mu] = C\ , \qquad (4)$$

where C is a Hermitian matrix, and consider an identity

$$1 = \int_H dg\, \delta\left(\, i\,[X_\mu, A_\mu^g] - C\,\right) \Delta(A_\mu)\ , \qquad (5)$$

where the Faddeev-Popov determinant $\Delta(A_\mu)$ needs to be evaluated for $A_\mu = X_\mu$ at the one-loop level. Making an infinitesimal transformation $g = 1 + i\,h$ on $A_\mu = X_\mu$ in (4), we obtain $[X_\mu,[X_\mu,h]] = C$. The linear transformation defined by the left hand side has zero eigenvalues for h belonging to the Lie algebra of $\prod_{i=1}^{s}\mathrm{U}(k_i)$. We should therefore restrict the matrix C, as well as h, to be within the tangent space of H. Since $\Delta(X_\mu)$ is given by the product of the eigenvalues of the above linear transformation in the restricted space, we obtain

$$\Delta(X_\mu) = \prod_{i,j=1}^{s}\ \prod_{l=|n_i-n_j|/2}^{(n_i+n_j)/2-1}{}' \left[\chi^2\, l\,(l+1)\right]^{k_i k_j (2l+1)} , \qquad (6)$$

where the symbol $\prod{}'$ implies that $l = 0$ is excluded. Inserting the unity (5) and exploiting the SU(N) invariance, we rewrite the partition function as

$$Z = \mathrm{vol}(H)\int dA\, \delta(i[X_\mu, A_\mu] - C)\,\Delta(A_\mu)\, e^{-S}\ , \qquad (7)$$

where $\mathrm{vol}(H) = \mathrm{vol}(\mathrm{U}(N))/\prod_{i=1}^{s}\mathrm{vol}(\mathrm{U}(k_i))$ can be obtained by $\mathrm{vol}(\mathrm{U}(n)) = \frac{(2\pi)^{\frac{n(n+1)}{2}}}{(n-1)!\cdots 0!}$. Since the result does not depend on C, we integrate over it within the tangent space of H with the Gaussian weight $\mathcal{N} e^{-\frac{1}{2}N\alpha^4 \mathrm{tr}\, C^2}$, where $\mathcal{N} = \left(\frac{N\alpha^4}{2\pi}\right)^{\frac{1}{2}\{N^2 - \sum_{i=1}^{s}(k_i)^2\}}$. This yields an extra term $S_{\rm g.f.} = -\frac{1}{2}\,N\,\alpha^4\,\mathrm{tr}\,[X_\mu, A_\mu]^2$ in the action, which lifts the flat directions as desired.

Now we are ready to perform the integration over A_μ. Decomposing the variables as $A_\mu = X_\mu + \tilde{A}_\mu$, we expand the total action $S_{\rm tot} = S + S_{\rm g.f.}$ with respect to the fluctuation $\tilde{A}_\mu$ as

$$S_{\rm tot} = S_{\rm cl} + \frac{1}{2}\,N\,\alpha^4\,\mathrm{tr}\left(\tilde{A}_\mu\,\mathcal{Q}_{\mu\nu}\,\tilde{A}_\nu\right) + \cdots\ , \qquad (8)$$

where the operator $\mathcal{Q}_{\mu\nu}$ is given by

$$\mathcal{Q}_{\mu\nu} = \left\{\chi^2\,(\mathcal{L}_\lambda)^2 + \rho^2\right\}\delta_{\mu\nu} - i\,\rho^2\,\epsilon_{\mu\nu\lambda}\,\mathcal{L}_\lambda\ . \qquad (9)$$

We have introduced an operator $\mathcal{L}_\lambda$ which acts on a $N\times N$ matrix M as $\mathcal{L}_\lambda\, M = [L_\lambda, M]$. The remaining task is to solve the eigenvalue problem $\mathcal{Q}_{\mu\nu}\tilde{A}_\nu = \lambda\,\tilde{A}_\mu$. Since L_μ has a block-diagonal form, we can decompose the matrix $\tilde{A}_\mu$ into blocks of size $n_i \times n_j$, and it suffices to solve the eigenvalue problem within each block [20]. This can be done in a similar way as in the BMN matrix model [21]. As a complete basis for each block, we choose the eigenstates $|\, l, m\rangle$ of $(\mathcal{L}_\lambda)^2$ and $\mathcal{L}_3$ with the eigenvalues $l\,(l+1)$ and m, where $|m| \le l$ and $|n_i - n_j|/2 \le l \le (n_i+n_j)/2 - 1$. Expanding each block of $\tilde{A}_\mu$ as

$$(\tilde{A}_\mu)_{n_i\times n_j} = \sum_{l=|n_i-n_j|/2}^{(n_i+n_j)/2-1}\ \sum_{m=-l}^{l} \tilde{A}_\mu^{(lm)}\,|\, l, m\rangle \qquad (10)$$

and using the properties

$$(\mathcal{L}_1 \pm i\mathcal{L}_2)\,|\, l, m\rangle = b_\pm\,|\, l, m\pm 1\rangle\ , \qquad (11)$$

$$b_\pm = \sqrt{\,l\,(l+1) - m\,(m\pm 1)}\ , \qquad (12)$$

we obtain the eigenvalue equation within each block as

$$\{\Lambda - \rho^2(1-m)\}\tilde{A}_+^{(l,m)} = \rho^2 b_- \tilde{A}_3^{(l,m-1)} , \quad (13)$$
$$\{\Lambda - \rho^2(1+m)\}\tilde{A}_-^{(l,m)} = -\rho^2 b_+ \tilde{A}_3^{(l,m+1)} , \quad (14)$$
$$\{\Lambda - \rho^2\}\tilde{A}_3^{(l,m)} = \frac{1}{2}\rho^2 (b_+ \tilde{A}_+^{(l,m+1)} - b_- \tilde{A}_-^{(l,m-1)}) , \quad (15)$$

where we have introduced $\tilde{A}_\pm^{(l,m)} = \tilde{A}_1^{(l,m)} \pm i\, \tilde{A}_2^{(l,m)}$ and $\Lambda = \lambda - \chi^2\, l\,(l+1)$. For $l \geq 1/2$ we have eigenvalues

$$\lambda_1(l) = \chi^2\, l\,(l+1) , \quad (16)$$
$$\lambda_2(l) = \chi^2\, l\,(l+1) - \rho^2\, l , \quad (17)$$
$$\lambda_3(l) = \chi^2\, l\,(l+1) + \rho^2\,(l+1) , \quad (18)$$

whose degeneracy is $2\,l+1$, $2\,l-1$, $2\,l+3$, respectively. (For $l = 1/2$ the 2nd eigenvalue does not appear.) For $l = 0$ we have $\lambda = \rho^2$ with 3-fold degeneracy. Thus the integration over $\tilde{A}_\mu$ yields

$$\mathcal{Z} = \left(\frac{2\pi}{N\alpha^4}\right)^{\frac{3}{2}N^2} \cdot \prod_{i=1}^{s} \rho^{-3(k_i)^2} \cdot \prod_{i,j=1}^{s} (q_{n_i n_j})^{k_i k_j} , \quad (19)$$

$$q_{nm} = \prod_{l=|n-m|/2}^{(n+m)/2-1}{}' \left[\lambda_1(l)^{2l+1}\lambda_2(l)^{2l-1}\lambda_3(l)^{2l+3}\right]^{-\frac{1}{2}} . \quad (20)$$

Bringing all the factors together, we get

$$Z = \mathrm{vol}(H)\, \mathcal{N}\, \Delta(X_\mu)\, \mathcal{Z}\, e^{-S_{\mathrm{cl}}} . \quad (21)$$

In fact this result holds also for the case $X_\mu = 0$ formally.

If we take the large-N limit of the free energy $F = -\log Z$ with fixed α and ρ, the one-loop terms give maximally $\mathrm{O}(N^2 \log N)$ contributions. The $\mathrm{O}(N^2 \log N)$ terms coming from $\mathrm{vol}(H)$ and $\mathcal{N}$ cancel each other in general. Apart from the universal term $\frac{3}{2}N^2 \log N$ coming from the first factor in (19), we have contributions from $\Delta(X_\mu)$ and the third factor in (19), which induce extra $\mathrm{O}(N^2 \log N)$ terms when n_i becomes of $\mathrm{O}(N)$.

The $\rho < \frac{2}{3}$ regime.— Since the single fuzzy sphere has a *negative* classical action of $\mathrm{O}(\alpha^4 N^4)$ in this regime, no other solutions can be the true vacuum unless α becomes as small as $\mathrm{O}(\frac{1}{\sqrt{N}})$. The free energy for the single fuzzy sphere $A_\mu = \chi\, L_\mu^{(N)}$ is obtained as

$$\frac{1}{N^2} F_{\mathrm{FS}} = \frac{1}{4} N^2 \alpha^4 f(\chi) + \frac{5}{2}\log N + 4\,\log\alpha - \delta \quad (22)$$

at large N, where δ is an $\mathrm{O}(1)$ constant which depends only on ρ. (The one-loop calculation is reliable for the single fuzzy sphere as far as $\alpha\sqrt{N}$ is large [11,12]. This is not the case for all the solutions.)

On the other hand, at small α we find, after rescaling $\mathcal{A}_\mu = \alpha A_\mu$, that the quartic term in the action (1) becomes dominant, and obtain the so-called Yang-Mills phase [11], which is described by the vacuum of the pure Yang-Mills model [22]. Using the result of the Gaussian expansion method [23], we obtain the free energy of this phase as

$$\frac{1}{N^2} F_{\mathrm{YM}} = \frac{3}{2}\,\log N + 3\,\log\alpha + \gamma \quad (23)$$

at large N, where $\gamma \sim -4.5$.

Comparing (22) and (23), we find that F_{YM} becomes smaller than F_{FS} at

$$\alpha < \alpha_{\mathrm{cr}} \equiv \frac{1}{\sqrt{N}} \left(\frac{2\,\log N}{|f(\chi)|}\right)^{1/4} . \quad (24)$$

This is consistent with the Monte Carlo simulations at $\rho = 0$ [11], where a first order phase transition has been observed with the upper and lower critical points $\alpha_{\mathrm{cr}}^{(\mathrm{u})} \sim 0.66$ and $\alpha_{\mathrm{cr}}^{(\mathrm{l})} \sim \frac{2.1}{\sqrt{N}}$, respectively. Notice the inequality $\alpha_{\mathrm{cr}}^{(\mathrm{l})} < \alpha_{\mathrm{cr}}(\rho = 0) < \alpha_{\mathrm{cr}}^{(\mathrm{u})}$.

The $\rho > \frac{2}{3}$ regime.— Since $f(\chi) > 0$ in this regime, in order for some nontrivial solution to have smaller free energy than the $A_\mu = 0$ solution, it should have a classical action of $\mathrm{O}(N^2)$ or of smaller order. Thus it turns out to be sufficient to consider the case where s and all the n_i are of $\mathrm{O}(1)$, while k_i are of $\mathrm{O}(N)$. Such a configuration (except $A_\mu = 0$) is analogous to the ones that are interpreted [24] in the BMN model as a collection of coincident transverse 5-branes in M-theory. Let us denote $k_i = r_i\, N$, where r_i are real parameters of $\mathrm{O}(1)$. In what follows we consider the large-N limit taken in this way.

Let us first consider the case where α is very large, so that we can neglect all the α-independent terms in the free energy and obtain the relevant terms

$$\frac{1}{N^2} F \sim A \sum_{i=1}^{s} r_i\, n_i \Big\{(n_i)^2 - 1\Big\} + B \sum_{i=1}^{s} (r_i)^2 , \quad (25)$$

where $A = \frac{1}{4}\,\alpha^4\, f(\chi)$ and $B = 2\,\log\alpha$. We determine the r_i that minimize the free energy within the constraint $\sum_{i=1}^{s} n_i r_i = 1$ using the Lagrangian multiplier λ as

$$r_i = \frac{1}{2B}\left[\lambda\, n_i - A\, n_i \Big\{(n_i)^2 - 1\Big\}\right] , \quad (26)$$

where λ should be fixed by the constraint. We may assume $n_p = p$, and requiring $r_i \geq 0$, we obtain $s(s^2-1)(4s^2-1) < 60B/A$, where s should be taken to be the largest possible integer.

Thus we obtain the $A_\mu = 0$ solution in the large-α limit as expected, but as we go below the critical α determined by $\frac{A}{B} = \frac{2}{3}$, $\mathrm{O}(N)$ numbers of 2×2 blocks start to appear. If we decrease α further, larger and larger blocks appear with the specific proportion (26). This result is valid when ρ is very close to $\frac{2}{3}$ so that the coefficient $f(\chi)$ in A is small and the transitions take place at large α.

At moderate α, the one-loop terms which are independent of α can no longer be neglected. We search for the minimum of the free energy numerically restricting ourselves to the case $n_p = p$ $(p = 1, \cdots, s)$ with $s = 7$. In Fig. 1 we plot the result for $\rho = 0.7$. (The situation for other values of $\rho\,(> \frac{2}{3})$ is qualitatively the same.) Since

we obtain $r_7 = 0$ throughout the whole region of α, the above restriction is considered to be harmless. The minimum free energy obtained in this way is plotted in Fig. 2, where we also plot the result (23) for the Yang-Mills phase, which should be valid for small α, and expected to be smoothly connected to the $A_\mu = 0$ solution at $\alpha \sim$ O(1). In the $\rho > \frac{2}{3}$ regime, since the action (1) is positive definite, we can easily prove that the free energy should be a monotonously increasing (continuous) function of α. We therefore consider that higher loop corrections increase the free energy for the "5-brane" at $\alpha \sim$ O(1), where the Yang-Mills phase should take over.

Combining all the results obtained above, we arrive at the phase diagram depicted in Fig. 3.

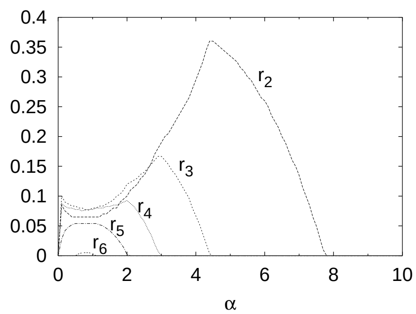

FIG. 1. The parameters r_i ($2 \le i \le 6$) that minimize the free energy are plotted against α for $\rho = 0.7$.

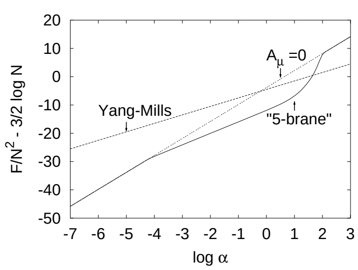

FIG. 2. The "free energy density" $\lim_{N\to\infty}(\frac{1}{N^2}F - \frac{3}{2}\log N)$ is plotted against $\log\alpha$ for $\rho = 0.7$. The solid line represents the result obtained by the one-loop calculation, which is valid at large α. The dip corresponds to some "5-brane" solution, and the straight part corresponds to the $A_\mu = 0$ solution. The dashed line represents the result (23) obtained for the Yang-Mills phase, which is valid at small α.

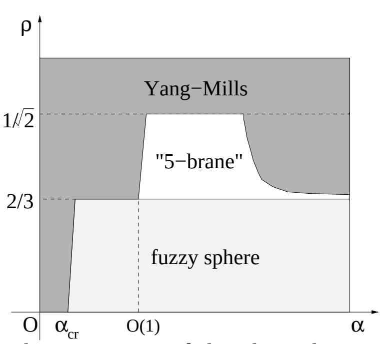

FIG. 3. A schematic view of the phase diagram of the massive Yang-Mills-Chern-Simons model. The critical point $\alpha_{\rm cr}$ between two phases below $\rho = \frac{2}{3}$ is given by eq. (24).

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